Section 6.3
Antidifferentiation by Parts

Product Rule for Differentiation

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

Product Rule in Integral Form

$$\int\left(u\frac{dv}{dx}\right)dx = \int\left(\frac{d}{dx}uv\right)dx - \int\left(v\frac{du}{dx}\right)dx$$

Integration by Parts Formula

$$\int u\,dv = uv - \int v\,du$$

Example

Evaluate $\int x \cos x \, dx =$

$$\int x\, e^{x}\, dx =$$

$$\int x^2 \ln x \, dx =$$

$$\int \ln x \, dx =$$

$$\int dx = \int 1 \, dx = x$$

$$u = \ln x \qquad dv = dx$$

$$du = \frac{1}{x} dx \qquad v = x$$

$$= x \ln x - \int x \cdot \frac{1}{x} dx$$

$$= x \ln x - \int 1 \, dx$$

$$= x \ln x - x + C$$

$$\int x^2 e^x \, dx =$$

$u = x^2 \quad dv = e^x dx$

$du = 2x\,dx \quad v = e^x$

$$= x^2 e^x - \int e^x \, 2x \, dx$$

$u = 2x \quad dv = e^x dx$

$du = 2\,dx \quad v = e^x$

$$= x^2 e^x - \left(2x e^x - \int e^x \, 2 \, dx \right)$$

$$= x^2 e^x - 2x e^x + 2e^x + C$$

The Order

L I P E T

LOG INV TRIG POLY EXP TRIG

Solve the differential equation $dy/dx = x \ln(x)$ subject to the initial condition $y = -1$ when $x = 1$. Confirm the solution graphically by showing that it conforms to the slope of the field.

Evaluate

$$\int e^x \cos x \, dx$$

$u = e^x \qquad dv = \cos x \, dx$

$du = e^x \, dx \qquad v = \sin x$

$= e^x \sin x - \int \sin x \, e^x \, dx \qquad u = e^x \quad dv = \sin x \, dx$

$du = e^x dx \quad v = -\cos x$

$= e^x \sin x - \left(-e^x \cos x - \int -\cos x \, e^x \, dx \right)$

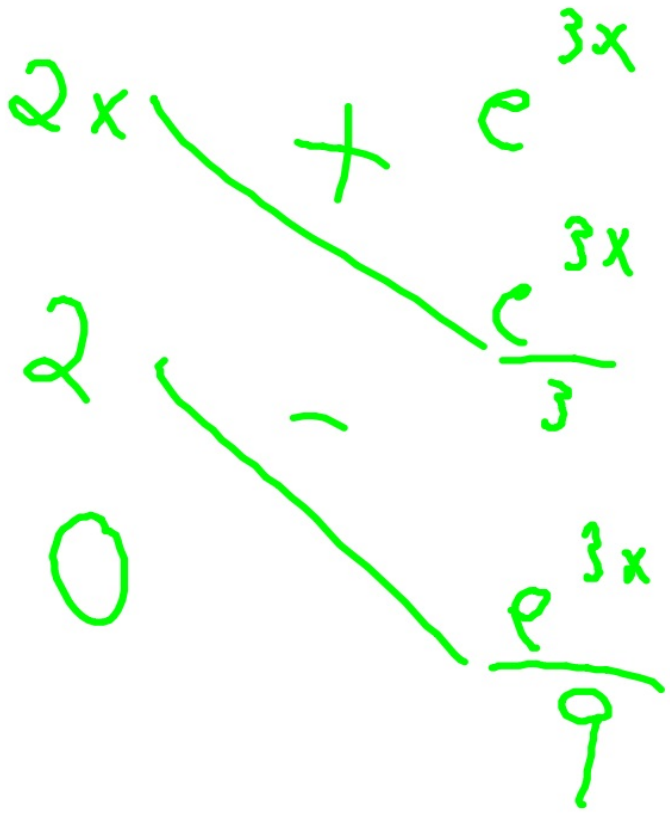
$\int e^x \cos x \, dx = e^x \sin x + e^x \cos x - \int e^x \cos x \, dx$

$+ \int e^x \cos x \, dx$

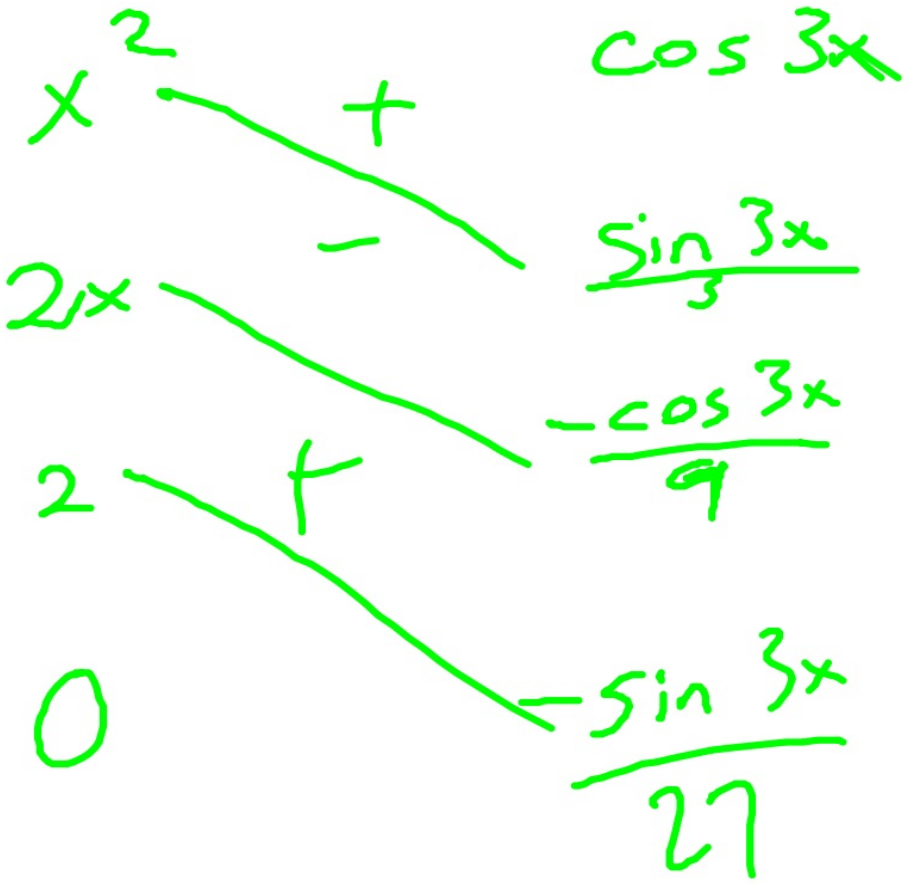
$+ \int e^x \cos x \, dx$

$$\int 2x\, e^{3x}\, dx$$

$\frac{2}{3}xe^{3x} - \frac{2}{9}e^{3x} + C$



$$\int x^2 \cos 3x \, dx$$



$$\int e^{2x} \sin x \, dx =$$

$u = e^{2x}$ $\quad dv = \sin x \, dx$

$du = 2e^{2x} dx$ $\quad v = -\cos x$

$$= -e^{2x}\cos x - \int -\cos x \, 2e^{2x} dx$$

$$= -e^{2x}\cos x + 2\int \cos x \, e^{2x} dx$$

$$\int x \csc^2 x \, dx$$

$$\int e^{3x} \cos x \, dx =$$

$$\int e^x \cos 2x \, dx =$$

Tabular Integration

- Shortcut for Integration by Parts

Only if u is a polynomial

Evaluate $\int x^2 e^x \, dx$

With $f(x) = x^2$ and $g(x) = e^x$, we list:

$f(x)$ and its derivatives		$g(x)$ and its integrals
x^2	$(+)$ →	e^x
$2x$	$(-)$ →	e^x
2	$(+)$ →	e^x
0	→	e^x

$$\int x^4 e^{3x}\, dx =$$

$$\int x^4 e^{(1/2)x}\, dx$$

$$\int x^3 \sin x \, dx$$

$$\int_{0}^{2} (x^2 - 5x)\, e^x \, dx =$$

$$\int \frac{x^2 - 9}{x} \, dx$$

$$\int \frac{x}{x^2 - 9}\, dx$$